Rational Pessimism, Rational Exuberance, and Markets for Macro Risks

Ravi Bansal

Fuqua School of Business

Duke University

Durham NC 27599-0120 USA

A. Ronald Gallant

Fuqua School of Business

Duke University

Durham NC 27599-0120 USA

George Tauchen

Department of Economics

Duke University

Durham NC 27599-0097 USA

By complicating the either the dynamics of the consumption endowment, the utility function, or both, the classical Lucus (1978) rational expectations model can be made to fit US annual macro data, 1921–2001, along the following dimensions:

- Pass tests of overidentifying restrictions.
- Match unconditional moments of consumption growth, dividendconsumption ratio, price-dividend ratio, stock returns, quadratic variation, equity premium, etc.
- Match the AR(1) coefficients of these same items.

Two such rational expectations models are estimated:

- Long run risks (Bansal and Yaron, 2002). Relies on low frequency movements in consumption growth, cash flows, and economic uncertainty to generate time-varying risk premia.
- Habit persistence (Campbell and Cochrane, 1999). Relies on time varying risk aversion to generate time-varying risk premia.

Features of the estimation:

- Cointegration of dividends and consumption imposed (LRR only).
- Cointegration of stock price and dividends imposed.
- Claims on macro aggregates proposed by Shiller (1998) are priced:
 - Puts and calls on consumption.
 - Puts and calls on wealth.

Dimensions along which the models differ:

- Models price options on wealth differently; they are far more expensive under the habit model.
- The price-dividend ratio is too tightly linked to past consumption in the habit model.
- Future stock returns are too tightly linked to the current price-dividend ratio in both models; more so in the habit model than the long run risk model.

Outline of Paper (www.duke.edu/~arg)

- Economic Models: LRR, SRR, HAB
 - Dynamics of the State Vector
 - Asset Pricing
- Time Aggregation: Monthly → Annual
- Data and Cointegrating Relationships
- Estimation Methodology
- Results

Conventions

```
t Time, in months

C_t The consumption endowment D_t Dividends on stocks

c_t = \log(C_t)

d_t = \log(D_t)
```

Consumption

$$c_t = c_{t-1} + \mu_c + x_{t-1} + \epsilon_{ct}$$

where

 $\mu_c + x_{t-1}$ is the expected growth rate of c

 x_t is a mean zero process

 ϵ_{ct} is a mean zero, iid process

Equivalently,

$$\Delta c_t = \mu_c + x_{t-1} + \epsilon_{ct}$$

where

$$\Delta c_t = c_t - c_{t-1}$$

The Consumption Growth Process

$$x_t = \rho_x x_{t-1} + \epsilon_{xt}$$

where

 ρ_x satisfies $0 < \rho_x < 1$

 ϵ_{xt} is a mean zero, iid process

This is the key assumption in the model. Consumption growth is not iid, as is often assumed, but rather varies according to a slowly mean reverting process that is masked by noise; a priori, $\rho_x \approx 1$.

Dividends

Dividends are presumed to be cointegrated with consumption

$$d_t - c_t = \mu_{dc} + s_t$$

where

 μ_{dc} is a constant, $D_t/C_t \approx e^{\mu_{dc}}$

 s_t is a mean zero, I(0) process

Assume

$$s_t = \rho_s s_{t-1} + \lambda_{sx} x_{t-1} + \epsilon_{st}$$

where

 ρ_s satisfies $0 < \rho_x < 1$

 ϵ_{st} is a mean zero, iid process

Dividend Growth

$$\Delta d_t = \mu_c + (\rho_s - 1)s_{t-1} + (1 + \lambda_{sx})x_{t-1} + \epsilon_{st} + \epsilon_{ct}$$

A priori, $\lambda_{sx}\approx 2$, $\rho_{s}\approx 1$. For these values, the fluctuations in dividend growth are more extreme than the fluctuations in consumption growth, which allows a risk premium without requiring extreme values of the risk aversion parameter.

Dynamics as a VAR

$$\Delta c_t = \mu_c + x_{t-1} + \epsilon_{ct}$$

$$s_t = \rho_s s_{t-1} + \lambda_{sx} x_{t-1} + \epsilon_{st}$$

$$x_t = \rho_x x_{t-1} + \epsilon_{xt}$$

Dynamics of the State Vector Stochastic Volatility

$$\epsilon_t = \operatorname{diag}(b_t) \, \Psi \, \eta_t$$

$$b_t = \begin{pmatrix} \exp(b_c + b_{cc} \, \nu_t) \\ \exp(b_s + b_{ss} \, \nu_t) \\ \exp(b_x + b_{xx} \, \nu_t) \end{pmatrix}$$

$$\nu_t = \mu_\sigma + \rho_\sigma \nu_{t-1} + \epsilon_{\sigma t}$$

where

 u_t is the common volatility shock

 ρ_{σ} satisfies $0 < \rho_x < 1$

 $\epsilon_{\sigma t}$ is a mean zero, iid process

 Ψ is an upper triangular matrix

 η_t is mean zero, unit variance, iid

The State Vector

$$u_t = \left(\begin{array}{c} s_t \\ x_t \\ \nu_t \end{array}\right)$$

Conventions

```
Time, in months
 t
C_t
         The consumption endowment
D_t
         Dividends on stocks
P_{ct}
         Price of asset paying the endowment
P_{dt}
         Price of stocks (market capitalization)
v_{ct} = P_{ct}/C_t (endowment price dividend ratio)
     = P_{dt}/D_t (stock price dividend ratio)
         Return on the endowment
r_{ct}
         Return on the dividend stream
r_{dt}
         One-step-ahead risk-free rate
r_{ft}
```

Epstein-Zin-Weil Utility Function

$$U_t = \left[(1 - \delta) C_t^{\frac{1 - \gamma}{\theta}} + \delta \left(\mathcal{E}_t U_{t+1}^{1 - \gamma} \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1 - \gamma}}$$

where

 γ is the coefficient of risk aversion

 ψ is the elasticity of intertemporal substitution

and

$$\theta = \frac{1 - \gamma}{1 - 1/\psi}$$

Return on the Endowment Stream

$$v_{ct} = \mathcal{E}_t \left\{ \delta^{\theta} \exp[-(\theta/\psi)\Delta c_{t+1} + (\theta - 1)r_{c,t+1}] \right.$$
$$\left. \times (1 + v_{c,t+1}) \exp(\Delta c_{t+1}) \right\}$$

$$r_{ct} = \log \left[\frac{1 + v_{ct}}{v_{c,t-1}} \exp(\Delta c_t) \right]$$

Return on the Dividend Stream

$$v_{dt} = \mathcal{E}_t \left\{ \delta^{\theta} \exp[-(\theta/\psi)\Delta c_{t+1} + (\theta - 1)r_{c,t+1}] \right.$$
$$\left. \times (1 + v_{d,t+1}) \exp(\Delta d_{t+1}) \right\}$$

$$r_{dt} = \log \left[\frac{1 + v_{dt}}{v_{d,t-1}} \exp(\Delta d_t) \right]$$

Risk Free Rate

$$e^{-r_{ft}} = \mathcal{E}_t \left\{ \delta^{\theta} \exp[-(\theta/\psi)\Delta c_{t+1} + (\theta - 1)r_{c,t+1}] \right\}$$

Implied MRS

$$M_{t,t+1} = \delta^{\theta} \exp[-(\theta/\psi)\Delta c_{t+1} + (\theta - 1)r_{c,t+1}]$$

General Payoff

 P_{gt} is the price at time t of the payoff G_{t+k} at time t+k

$$P_{gt} = \mathcal{E}_t \left[\left(\prod_{j=1}^k M_{t+j-1,t+j} \right) G_{t+k} \right]$$

This expression can be used to compute the yield curve and to price any cash flow generated within the model such as put and call options on consumption and wealth.

Model Variants: LRR and SRR

Long Run Risks (LRR)

Model as described above.

Short Run Risks (SRR)

Similar to Hall (1978); c_t is a random walk with drift and there is no stochastic volatility. Accomplished by setting selected parameters above to zero. Implies constant risk premia.

Habit Persistence Model (HAB)

Campbell-Cochrane (1999) Utility

$$U_{t} = \mathcal{E}_{t} \sum_{i=0}^{\infty} \delta^{i} \frac{(C_{t+i} - X_{t+i})^{1-\gamma} - 1}{1-\gamma}$$

where X_t is the habit stock that evolves as follows

$$h_{t} = \log \frac{C_{t} - X_{t}}{C_{t}} \quad (\text{log surplus ratio})$$

$$h_{t+1} = (1 - \rho_{h})\bar{h} + \rho_{h}h_{t} + \lambda(h_{t})\epsilon_{c,j+1}$$

$$\lambda(h_{t}) = \begin{cases} \frac{1}{\bar{H}}\sqrt{1 - 2(h_{t} - \bar{h})} - 1 & h_{t} \leq h_{max} \\ 0 & h_{t} > h_{max} \end{cases}$$

$$\bar{H} = \sigma_{\epsilon_{c}}\frac{\gamma}{\phi_{h}} \quad h_{max} = \bar{h}\frac{1}{2}(1 - \bar{H}^{2})$$

Habit Persistence Model (HAB)

Campbell-Cochrane Dynamics

For programming convenience, implemented as a simplification of LRR dynamics:

$$d_t = \mu_{dc} + c_t + s_t$$

$$\Delta c_t = \mu_c + \epsilon_{ct}$$

$$\Delta d_t = \mu_c + \epsilon_{ct} + \epsilon_{dt}$$

where $\rho_s = 1$ so that d_t and c_t are not cointegrated.

The State Vector

$$u_t = \left(\begin{array}{c} h_t \\ \lambda(h_t) \end{array}\right)$$

Asset Pricing as Above

Data

Conventions

```
\begin{array}{ll} t & = & 12, \, 24, \, \dots, \, 864 \, = \, 12 \times 72 \, = \, n \\ \\ P^a_{dt} & \text{end-of-year per capita stock market value} \\ D^a_t & \text{annual aggregate per capita dividend} \\ C^a_t & \text{annual per capita consumption} \\ r^a_{dt} & \text{annual real geometric return} \\ Q^a_t & \text{annual quadratic variation} \end{array}
```

Data

Sources

Annual observations 1929–2001; 72 years.

Financial market variables are monthly data from CRSP converted to real (\$1996) per capita then annualized.

The consumption endowment is annual real (\$1996) per capita consumption, nondurables and services, Bureau of Economic Analysis Web site.

Details:

Mid-year population data and the monthly CPI are from BEA.

Monthly (real per capita) stock market valuation series is the month-end capitalizations of NYSE+AMEX converted to \$1996 using monthly CPI; annual is the year-end value divided by BEA population.

The monthly (real per capita) dividend return is the difference between the value weighted return and capital return on the NYSE+AMEX; this times the preceding month's market capitalization is converted to a real (\$1996) monthly dividend series using the monthly CPI; annual is this series aggregated over the year divided by the BEA population.

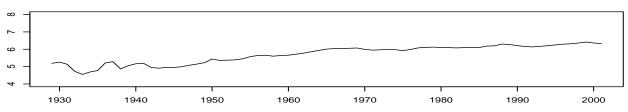
Monthly (real per capita) stock return is value weighted return on NYSE+AMEX converted to real using the CPI; annual geometric return is this return cumulated over the year.

Annual quadratic variation is the sum of the monthly squared real geometric returns.

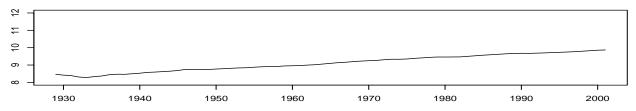
Log Real Per Capita Stock Market Value



Log Real Per Capita Stock Market Dividend



Log Real Per Capita Consumption, Measured at Annual Frequency



Real Annual Geometric Stock Market Returns



Log Quadratic Variation of Real Stock Market Returns



Cointegrating Relationships

 $p_{dt}^a - d_t^a = I(0)$ Well documented in the literature

 $d_t^a - c_t^a = I(0)$ Verified with a reduced rank regression

 $c_t^a - c_{t-12}^a = I(0)$ Well documented in the literature

Data to be Confronted by the Model

Used for estimation

$$y_{t} = \begin{pmatrix} d_{t}^{a} - c_{t}^{a} \\ c_{t}^{a} - c_{t-12}^{a} \\ p_{dt}^{a} - d_{t}^{a} \\ r_{dt}^{a} \end{pmatrix}$$

Used for model validatation

$$y_{t} = \begin{pmatrix} d_{t}^{a} - c_{t}^{a} \\ c_{t}^{a} - c_{t-12}^{a} \\ p_{dt}^{a} - d_{t}^{a} \\ r_{dt}^{a} \\ q_{t}^{a} \end{pmatrix}$$

$$t = 12, 24, 36, \dots, 12 \times 72$$

Model Solution Method

Observation

We use a simulation estimator which means that a long simulation of the state vector $\{\hat{u}_t\}_{t=1}^N$ will be available for each value of the parameters

$$\rho_{(1)} = (\mu_c, \lambda_{cs}, \rho_s, \lambda_{sx}, \rho_x, b_{cc}, b_{ss}, b_{xx}, \Psi, \rho_\sigma, \mu_{dc})$$

considered during estimation. Since the simulation is available, one might as well use it to solve the model.

The Relevant Fact

An unconditional expectation of a random variable of the form $g(u_{t+1}, u_t)$ can be computed from a simulation as

$$\mathcal{E}(g) \doteq \frac{1}{N} \sum_{t=1}^{N} g(\hat{u}_{t+1}, \hat{u}_t)$$

to any desired degree of accuracy by taking N sufficiently large.

Model Solution Method

The Problem

Given a value of the parameters of the state dynamics

$$\rho_{(1)} = (\mu_c, \lambda_{cs}, \rho_s, \lambda_{sx}, \rho_x, b_{cc}, b_{ss}, b_{xx}, \Psi, \rho_\sigma, \mu_{dc})$$

and a value of the parameters of the utility function

$$\rho_{(2)} = (\delta, \gamma, \psi)$$

find the pricing function $v_c(u)$ that solves

$$\mathcal{E}\left\{v_c(u_t) - M(u_{t+1}, u_t)[1 + v_c(u_{t+1})] \exp(\Delta c_{t+1}) \mid u_t = u\right\} = 0$$

for all $u \in \Re^3$, where

$$M(u_{t+1}, u_t) = \delta^{\theta} \exp[-(\theta/\psi)\Delta c_{t+1} + (\theta - 1)r_c(u_{t+1}, u_t)]$$

$$r_c(u_{t+1}, u_t) = \log \left[\frac{1 + v_c(u_{t+1})}{v_c(u_t)} \exp(\Delta c_{t+1}) \right].$$

Model Solution Method

GMM Solution Strategy

Assume $v_c(u)$ is adequately approximated by a quadratic

$$v_c(u) \doteq a_0 + a_1'u + u'A_2u$$

put

$$g(u_{t+1}, u_t) = \{v_c(u_t) - M(u_{t+1}, u_t)[1 + v_c(u_{t+1})] \exp(\Delta c_{t+1})\} Z_t$$

where $Z_t = [1, u_t', \text{vech}'(u_t, u_t)]'$ and solve the unconditional moment conditions

$$0 = \mathcal{E}g \doteq \frac{1}{N} \sum_{t=1}^{N} g(\hat{u}_{t+1}, \hat{u}_t)$$

for $\alpha = [a_0, a'_1, \operatorname{vech}'(A_2)]$

GMM or NL2SLS software can be used to solve this system.

Similarly for $v_d(u)$, and $r_f(u)$.

Estimation Method

Efficient Method of Moments (EMM) with a one lag VAR auxiliary model. With this choice of auxiliary model, the estimator is asymptotically equivalent to the estimator proposed by Tony Smith (1993).

Ex Ante Risk Free Rate

- The ex ante real rate risk free rate of interest is not directly observable but any reasonable asset pricing model must accommodate the evidence that the risk free rate r_{ft} is about 0.896 percent per annum with low volatility (Campbell, 2002).
- ullet We impose this restriction on $\mathcal{E}(r_{ft})$ by imposing

$$\frac{1}{N} \sum_{t=1}^{n} r_{ft} = 0.000743618 \pm 0.0004167$$

on the monthly simulation.

• It binds. Without it, the estimation method chooses a risk free rate on the same order of magnitude as the real stock return. With it, estimates imply a reasonable equity premium.

Models Considered

SRR, LRR, HAB

Selected parameters calibrated, remainder estimated with EMM.

Auxiliary Model

The auxiliary model is an uncoupled VAR with one lag fit to the data without quadratic variation.

Model Evaluation

When evaluating the performance of Models SRR, LRR, and HAB, quadratic variation is taken into account.

Parameter Estimates

	SRR	Model	LRR N	Model
Parameter	Estimate	SE	Estimate	SE
$egin{array}{l} \mu_c \ ho_s \ \lambda_{sx} \ ho_x \ b_{cc} \ b_{ss} \ b_{xx} \ \psi_{cc} \end{array}$	0.002178 0.9892 0.00646	0.000290 0.0080 0.00458	0.001921 0.9583 2.5004 0.9871 0.1432 0.8188 0.1100 0.0034	0.000332 0.1032 16.9346 0.0088 c 0.2788 c
$\psi_{cs} \ \psi_{ss} \ \psi_{xx} \ ho_{\sigma}$	0.00146 0.02487	0.01746 0.00809	.4109e-06 0.000120 0.9866	.2203e-05 c 0.0011
$egin{array}{c} \delta \ heta \ \psi \ \gamma \end{array}$	0.997488 -195.79 2.00 98.8969	0.004369 233.59 c 116.7968	0.999566 -12.2843 2.00 7.1421	0.000343 7.6243 c 3.8122
μ_{dc}	-3.3965	0.0428	-3.3857	0.0540
	$\chi^2(5) = 41$.051 (.9e-7)	$\chi^2(3) = 10.50$	01 (0.0148)

Parameter Estimates

	HAB Model						
Parameter	Estimate	SE					
μ_c	0.001937	0.000334					
$\psi_{cc}\ \psi_{cs}\ \psi_{ss}$	0.003578 -0.003329 0.002187	0.013025 0.014111 0.000290					
$ ho_h$	0.9889	0.0099					
$rac{\delta}{\gamma}$	0.9954 1.0357	0.0042 2.1325					
μ_{dc}	-2.8237	0.0658					
	$\chi^2(5) = 14.4$	76 (0.0129)					

Unconditional Moments, LRR

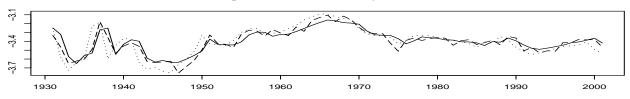
		Observed		Predict	ted- LRR
		Mean	Std Dev	Mean	Std Dev
Log dividend consumption ratio	$d_t^a-c_t^a$	-3.399	0.162	-3.384	0.154
Consumption growth (% Per Year)	$100(imes c_t^a - c_{t-12}^a)$	1.95	2.24	2.34	2.36
Price dividend ratio	$\exp(v_{dt}^a)$	28.24	12.08	27.47	8.01
Return(% Per Year), dividend	$100 imes r_{dt}^a$	6.02	19.29	6.29	16.00
$100 imes \sqrt{ ext{Quadratic variation}}$	$100 \times std^a_t$	16.69	09.32	14.95	8.21
Risk free rate (% Per Year)	$100 imes r_{ft}^a$			0.78	1.35
Return (% Per Year), consumption	$100 imes r_{ct}^a$			2.34	3.95
Equity premium (% Per Year)	$100 imes r_{dt}^a - r_{ft}^a$			5.51	16.09
	v				

Unconditional Moments, HAB

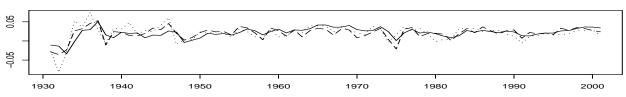
		Observed		Predict	ed- HAB
		Mean	Std Dev	Mean	Std Dev
Log dividend consumption ratio	$d_t^a - c_t^a$	-3.399	0.162	-3.385	0.161
Consumption growth(% Per Year)	$100(\times c^a_t - c^a_{t-12})$	1.95	2.24	2.35	1.38
Price dividend ratio	$exp(v_{dt}^a)$	28.24	12.08	29.08	7.32
Return(% Per Year), dividend	100 $ imes r^a_{dt}$	6.02	19.29	6.22	17.72
$100 imes \sqrt{ extsf{Quadratic variation}}$	$100 \times std^a_t$	16.69	09.32	15.56	9.88
Risk free rate (% Per Year)	100 $ imes$ r^a_{ft}			1.23	1.91
Return(% Per Year), consumption	$100 imes r_{ct}^a$			6.29	18.12
Equity premium(% Per Year)	$100 imes r_{dt}^a - r_{ft}^a$			4.99	17.72

One Step Ahead Forecasts, LRR Model

Log of Dividend Consumption Ratio



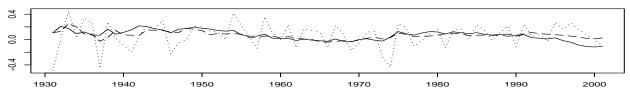
Consumption Growth



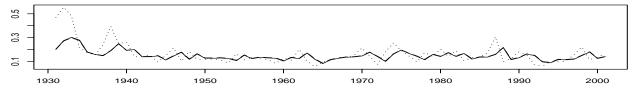
Log of Price Dividend Ratio



Annual Equity Returns

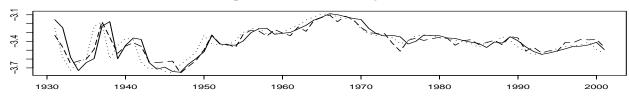


Equity Returns Volatility

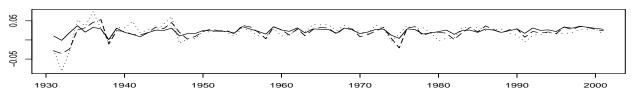


One Step Ahead Forecasts, HAB Model

Log of Dividend Consumption Ratio



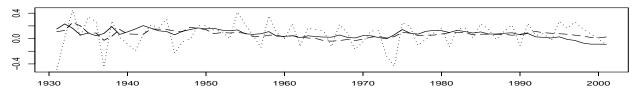
Consumption Growth



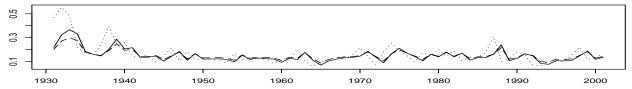
Log of Price Dividend Ratio



Annual Equity Returns



Equity Returns Volatility



Average Put and Call Prices on the Stock

	Long Run Risks (LRR)				Lo	ong F	Run R	isks (HAB	3)		
		Call		Put		Call		Put				
Strike/Underlying:	0.99	1.00	1.01	0.99	1.00	1.01	0.99	1.00	1.01	0.99	1.00	1.01
Expiration (mo)												
1	2.19	1.65	1.22	1.38	1.85	2.41	2.21	1.67	1.27	1.44	1.89	2.50
2	2.79	2.28	1.84	2.18	2.67	3.23	2.83	2.30	1.88	2.28	2.75	3.33
3	3.25	2.75	2.31	2.83	3.33	3.89	3.30	2.78	2.35	2.97	3.45	4.01
4	3.61	3.13	2.69	3.38	3.89	4.45	3.68	3.17	2.72	3.56	4.04	4.59
5	3.93	3.45	3.01	3.88	4.40	4.96	4.00	3.49	3.04	4.10	4.58	5.13
6	4.21	3.73	3.30	4.34	4.86	5.42	4.29	3.78	3.33	4.61	5.09	5.63
7	4.46	3.99	3.56	4.76	5.29	5.85	4.55	4.05	3.59	5.08	5.57	6.11
8	4.69	4.23	3.80	5.15	5.68	6.25	4.79	4.29	3.83	5.54	6.03	6.56
9	4.90	4.44	4.01	5.54	6.07	6.64	5.01	4.51	4.06	5.98	6.48	7.01
10	5.08	4.63	4.21	5.92	6.45	7.02	5.21	4.72	4.26	6.41	6.90	7.44
11	5.25	4.80	4.38	6.28	6.82	7.39	5.40	4.90	4.45	6.81	7.31	7.84
12	5.40	4.96	4.54	6.64	7.19	7.76	5.57	5.07	4.62	7.22	7.71	8.24

Average Put and Call Prices on Consumption

	Long Run Risks (LRR)					2)	Long Run Risks (HAB)	
		Call		Put			Call Put	
Strike/Underlying:	0.99	1.00	1.01	0.99	1.00	1.01	0.99 1.00 1.01 0.99 1.00 1.0)1
Expiration (mo)								
1	0.38	0.31	0.24	0.12	0.14	0.18	0.35 0.28 0.22 0.09 0.13 0.1	.7
2	0.59	0.51	0.44	0.17	0.19	0.21	0.53 0.46 0.39 0.12 0.15 0.1	.9
3	0.79	0.71	0.63	0.20	0.22	0.24	0.69 0.62 0.55 0.13 0.16 0.1	.9
4	0.98	0.90	0.82	0.23	0.25	0.27	0.85 0.78 0.71 0.14 0.16 0.1	.9
5	1.16	1.08	1.00	0.25	0.27	0.29	1.01 0.93 0.86 0.14 0.16 0.1	.9
6	1.34	1.26	1.18	0.28	0.29	0.31	1.16 1.08 1.01 0.14 0.16 0.1	.9
7	1.51	1.43	1.35	0.30	0.31	0.33	1.31 1.23 1.16 0.14 0.16 0.1	.8
8	1.69	1.60	1.52	0.32	0.33	0.35	1.46 1.38 1.30 0.14 0.16 0.1	.8
9	1.86	1.77	1.69	0.34	0.35	0.37	1.61 1.53 1.45 0.14 0.16 0.1	.8
10	2.03	1.94	1.86	0.36	0.37	0.39	1.76 1.68 1.60 0.14 0.16 0.1	.8
11	2.19	2.11	2.03	0.38	0.40	0.41	1.91 1.82 1.74 0.14 0.16 0.1	.7
12	2.36	2.27	2.19	0.41	0.42	0.44	2.05 1.97 1.89 0.14 0.15 0.1	.7

Average Put and Call Prices on Wealth

	Long Run Risks (LRR)					Long	Run R	isks (HAB	3)	
		Call			Put		Call		Put		
Strike/Underlying:	0.99	1.00	1.01	0.99	1.00	1.01	0.99 1.0	00 1.01	0.99	1.00	1.01
Expiration (mo)											
1	1.18	0.43	0.10	0.11	0.36	1.03	2.26 1.7	72 1.32	1.49	1.95	2.55
2	1.35	0.63	0.22	0.22	0.50	1.09	2.90 2.3	38 1.96	2.36	2.84	3.41
3	1.51	0.80	0.35	0.32	0.61	1.15	3.39 2.8	38 2.44	3.07	3.55	4.11
4	1.65	0.95	0.47	0.40	0.70	1.21	3.78 3.2	27 2.82	3.68	4.17	4.71
5	1.79	1.09	0.59	0.48	0.78	1.27	4.12 3.6	3.16	4.23	4.72	5.26
6	1.92	1.23	0.71	0.55	0.85	1.32	4.41 3.9	91 3.45	4.75	5.24	5.78
7	2.05	1.36	0.82	0.62	0.92	1.38	4.68 4.1	18 3.73	5.24	5.74	6.27
8	2.18	1.48	0.93	0.68	0.98	1.43	4.93 4.4	13 3.97	5.71	6.20	6.74
9	2.30	1.61	1.04	0.75	1.05	1.48	5.15 4.6	66 4.21	6.17	6.66	7.20
10	2.42	1.72	1.15	0.81	1.12	1.54	5.36 4.8	37 4.42	6.61	7.10	7.64
11	2.53	1.84	1.26	0.88	1.18	1.60	5.55 5.0	06 4.61	7.02	7.52	8.05
12	2.65	1.95	1.37	0.96	1.26	1.66	5.73 5.2	24 4.78	7.43	7.93	8.46

Predictability Projections: Price Dividend Ratios

	Obse	rved	Predi	icted
	Coef	SE	LRR	HAB
Intercept Δc_t Δc_{t-24} Δc_{t-48} Δc_{t-60} Δc_{t-72} Δc_{t-84}	3.4168	0.0055	3.2520	2.5046
	-0.3021	0.1744	1.0716	9.8700
	-1.0560	0.1926	0.0383	5.5366
	-0.6209	0.1925	0.2521	5.8474
	-0.3480	0.1926	-0.0176	5.2568
	-0.1263	0.1926	-0.0442	3.7064
	-0.2006	0.1744	-0.0857	4.7340
R^2	0.0	36	0.011	0.529

Notes: Shown above are the linear projections of the log price dividend ratio, v_d on contemporaneous and five annual lags of log consumption growth Δc . The period for the observed projection is 1935–2001. The predicted values are from long simulations from the Long Run Risks LRR Model and the Habit Persistence HAB Model.

Long Horizon Predictability Projections: Cumulative Future Return on the Price Dividend Ratio

Horizon(Years)	Observed	LRR	HAB
1 2 3 4 5	0.038 0.060 0.071 0.071 0.070	0.129 0.172 0.202	0.112 0.207 0.289 0.362 0.423

Notes: The table shows R^2 's from projections of cumulative annual geometric returns for 1,2,...,5 years ahead onto the log price dividend ratio for the observed data, 1935–2001, and for long simulations from the Long Run Risks **LRR** Model and the Habit Persistence **HAB** Model.