Rational Pessimism, Rational Exuberance, and Markets for Macro Risks

Ravi Bansal
Fuqua School of Business
Duke University
Durham NC 27599-0120 USA

A. Ronald Gallant
Fuqua School of Business
Duke University
Durham NC 27599-0120 USA

George Tauchen
Department of Economics
Duke University
Durham NC 27599-0097 USA
Overview

By complicating the either the dynamics of the consumption endowment, the utility function, or both, the classical Lucus (1978) rational expectations model can be made to fit US annual macro data, 1921–2001, along the following dimensions:

- Pass tests of overidentifying restrictions.

- Match unconditional moments of consumption growth, dividend-consumption ratio, price-dividend ratio, stock returns, quadratic variation, equity premium, etc.

- Match the AR(1) coefficients of these same items.
Overview

Two such rational expectations models are estimated:

- Long run risks (Bansal and Yaron, 2002). Relies on low frequency movements in consumption growth, cash flows, and economic uncertainty to generate time-varying risk premia.

- Habit persistence (Campbell and Cochrane, 1999). Relies on time varying risk aversion to generate time-varying risk premia.
Overview

Features of the estimation:

- Cointegration of dividends and consumption imposed (LRR only).

- Cointegration of stock price and dividends imposed.

- Claims on macro aggregates proposed by Shiller (1998) are priced:
  - Puts and calls on consumption.
  - Puts and calls on wealth.
Overview

Dimensions along which the models differ:

- Models price options on wealth differently; they are far more expensive under the habit model.

- The price-dividend ratio is too tightly linked to past consumption in the habit model.

- Future stock returns are too tightly linked to the current price-dividend ratio in both models; more so in the habit model than the long run risk model.
Outline of Paper  (www.duke.edu/~arg)

- Economic Models: LRR, SRR, HAB
  - Dynamics of the State Vector
  - Asset Pricing
- Time Aggregation: Monthly $\rightarrow$ Annual
- Data and Cointegrating Relationships
- Estimation Methodology
- Results
LRR: Dynamics of the State Vector

Conventions

- $t$: Time, in months
- $C_t$: The consumption endowment
- $D_t$: Dividends on stocks
- $c_t = \log(C_t)$
- $d_t = \log(D_t)$
Dynamics of the State Vector

Consumption

\[ c_t = c_{t-1} + \mu_c + x_{t-1} + \epsilon_{ct} \]

where

\[ \mu_c + x_{t-1} \] is the expected growth rate of \( c \)
\[ x_t \] is a mean zero process
\[ \epsilon_{ct} \] is a mean zero, iid process

Equivalently,

\[ \Delta c_t = \mu_c + x_{t-1} + \epsilon_{ct} \]

where

\[ \Delta c_t = c_t - c_{t-1} \]
Dynamics of the State Vector
The Consumption Growth Process

\[ x_t = \rho x x_{t-1} + \epsilon_{xt} \]

where
\[ \rho_x \text{ satisfies } 0 < \rho_x < 1 \]
\[ \epsilon_{xt} \text{ is a mean zero, iid process} \]

This is the key assumption in the model. Consumption growth is not iid, as is often assumed, but rather varies according to a slowly mean reverting process that is masked by noise; a priori, \( \rho_x \approx 1 \).
Dynamics of the State Vector

Dividends

Dividends are presumed to be cointegrated with consumption

\[ d_t - c_t = \mu_{dc} + s_t \]

where

- \( \mu_{dc} \) is a constant,
- \( D_t/C_t \approx e^{\mu_{dc}} \)
- \( s_t \) is a mean zero, I(0) process

Assume

\[ s_t = \rho_s s_{t-1} + \lambda_s x_{t-1} + \epsilon_{st} \]

where

- \( \rho_s \) satisfies \( 0 < \rho_s < 1 \)
- \( \epsilon_{st} \) is a mean zero, iid process
Dynamics of the State Vector

Dividend Growth

\[ \Delta d_t = \mu_c + (\rho_s - 1)s_{t-1} + (1 + \lambda_{sx})x_{t-1} + \epsilon_{st} + \epsilon_{ct} \]

A priori, \( \lambda_{sx} \approx 2, \rho_s \approx 1 \). For these values, the fluctuations in dividend growth are more extreme than the fluctuations in consumption growth, which allows a risk premium without requiring extreme values of the risk aversion parameter.
Dynamics as a VAR

\[
\begin{align*}
\Delta c_t &= \mu_c + x_{t-1} + \epsilon_{ct} \\
s_t &= \rho_{ss} s_{t-1} + \lambda_{sx} x_{t-1} + \epsilon_{st} \\
x_t &= \rho_{sx} x_{t-1} + \epsilon_{xt}
\end{align*}
\]
Dynamics of the State Vector

Stochastic Volatility

\[ \epsilon_t = \text{diag}(b_t) \Psi \eta_t \]

\[ b_t = \begin{pmatrix}
\exp(b_c + b_{cc} \nu_t) \\
\exp(b_s + b_{ss} \nu_t) \\
\exp(b_x + b_{xx} \nu_t)
\end{pmatrix} \]

\[ \nu_t = \mu_{\sigma} + \rho_{\sigma} \nu_{t-1} + \epsilon_{\sigma t} \]

where

- \( \nu_t \) is the common volatility shock
- \( \rho_{\sigma} \) satisfies \( 0 < \rho_x < 1 \)
- \( \epsilon_{\sigma t} \) is a mean zero, iid process
- \( \Psi \) is an upper triangular matrix
- \( \eta_t \) is mean zero, unit variance, iid
The State Vector

\[ u_t = \begin{pmatrix} s_t \\ x_t \\ \nu_t \end{pmatrix} \]
Asset Pricing

Conventions

\( t \) \hspace{1cm} \text{Time, in months}

\( C_t \) \hspace{1cm} \text{The consumption endowment}

\( D_t \) \hspace{1cm} \text{Dividends on stocks}

\( P_{ct} \) \hspace{1cm} \text{Price of asset paying the endowment}

\( P_{dt} \) \hspace{1cm} \text{Price of stocks (market capitalization)}

\( v_{ct} = \frac{P_{ct}}{C_t} \) \hspace{1cm} \text{(endowment price dividend ratio)}

\( v_{dt} = \frac{P_{dt}}{D_t} \) \hspace{1cm} \text{(stock price dividend ratio)}

\( r_{ct} \) \hspace{1cm} \text{Return on the endowment}

\( r_{dt} \) \hspace{1cm} \text{Return on the dividend stream}

\( r_{ft} \) \hspace{1cm} \text{One-step-ahead risk-free rate}
Asset Pricing

Epstein-Zin-Weil Utility Function

\[ U_t = \left[ (1 - \delta) C_t^{1-\gamma} + \delta (E_t U_{t+1}^{1-\gamma})^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}} \]

where

- \( \gamma \) is the coefficient of risk aversion
- \( \psi \) is the elasticity of intertemporal substitution

and

\[ \theta = \frac{1 - \gamma}{1 - 1/\psi} \]
Asset Pricing

Return on the Endowment Stream

\[ v_{ct} = \mathcal{E}_t \left\{ \delta^\theta \exp\left[-\left(\frac{\theta}{\psi}\right)\Delta c_{t+1} + (\theta - 1)r_{c,t+1}\right] \times (1 + v_{c,t+1}) \exp(\Delta c_{t+1}) \right\} \]

\[ r_{ct} = \log \left[ \frac{1 + v_{ct}}{v_{c,t-1}} \exp(\Delta c_t) \right] \]
Asset Pricing

Return on the Dividend Stream

\[ v_{dt} = \mathcal{E}_t \left\{ \delta^\theta \exp\left[ -\left( \frac{\theta}{\psi} \right) \Delta c_{t+1} + (\theta - 1)r_{c,t+1} \right] \right. \]
\[ \times \left. (1 + v_{d,t+1}) \exp(\Delta d_{t+1}) \right\} \]

\[ r_{dt} = \log \left[ \frac{1 + v_{dt}}{v_{d,t-1}} \exp(\Delta d_t) \right] \]
Asset Pricing

Risk Free Rate

\[ e^{-r_{ft}} = \mathcal{E}_t\left\{ \delta^\theta \exp\left[ -(\theta/\psi)\Delta c_{t+1} + (\theta - 1)r_{c,t+1}\right] \right\} \]

Implied MRS

\[ M_{t,t+1} = \delta^\theta \exp\left[ -(\theta/\psi)\Delta c_{t+1} + (\theta - 1)r_{c,t+1}\right] \]
Asset Pricing

General Payoff

$P_{gt}$ is the price at time $t$ of the payoff $G_{t+k}$ at time $t + k$

$$P_{gt} = \mathcal{E}_t \left[ \left( \prod_{j=1}^{k} M_{t+j-1,t+j} \right) G_{t+k} \right]$$

This expression can be used to compute the yield curve and to price any cash flow generated within the model such as put and call options on consumption and wealth.
Model Variants: LRR and SRR

Long Run Risks (LRR)
Model as described above.

Short Run Risks (SRR)
Similar to Hall (1978); $c_t$ is a random walk with drift and there is no stochastic volatility. Accomplished by setting selected parameters above to zero. Implies constant risk premia.
Habit Persistence Model (HAB)

Campbell-Cochrane (1999) Utility

\[ U_t = E_t \sum_{i=0}^{\infty} \delta^i (C_{t+i} - X_{t+i})^{1-\gamma} \frac{1}{1-\gamma} \]

where \( X_t \) is the habit stock that evolves as follows

\[ h_t = \log \frac{C_t - X_t}{C_t} \] (log surplus ratio)

\[ h_{t+1} = (1 - \rho_h) \bar{h} + \rho_h h_t + \lambda(h_t) \epsilon_{c,j+1} \]

\[ \lambda(h_t) = \begin{cases} \frac{1}{\bar{H}} \sqrt{1 - 2(h_t - \bar{h}) - 1} & h_t \leq h_{max} \\ 0 & h_t > h_{max} \end{cases} \]

\[ \bar{H} = \sigma_{\epsilon_c} \frac{\gamma}{\phi_h} \]

\[ h_{max} = \bar{h} \frac{1}{2} (1 - \bar{H}^2) \]
Habit Persistence Model (HAB)

Campbell-Cochrane Dynamics
For programming convenience, implemented as a simplification of LRR dynamics:

\[
d_t = \mu_{dc} + c_t + s_t
\]

\[
\Delta c_t = \mu_c + \epsilon_{ct}
\]

\[
\Delta d_t = \mu_c + \epsilon_{ct} + \epsilon_{dt}
\]

where \( \rho_s = 1 \) so that \( d_t \) and \( c_t \) are not cointegrated.
The State Vector

\[ u_t = \begin{pmatrix} h_t \\ \lambda(h_t) \end{pmatrix} \]

Asset Pricing as Above
Data

Conventions

\[ t = 12, 24, \ldots, 864 = 12 \times 72 = n \]

- \( P_{dt}^a \): end-of-year per capita stock market value
- \( D_t^a \): annual aggregate per capita dividend
- \( C_t^a \): annual per capita consumption
- \( r_{dt}^a \): annual real geometric return
- \( Q_t^a \): annual quadratic variation
**Data Sources**

Annual observations 1929–2001; 72 years.

Financial market variables are monthly data from CRSP converted to real ($1996) per capita then annualized.

The consumption endowment is annual real ($1996) per capita consumption, nondurables and services, Bureau of Economic Analysis Web site.

**Details:**

Mid-year population data and the monthly CPI are from BEA.

Monthly (real per capita) stock market valuation series is the month-end capitalizations of NYSE+AMEX converted to $1996 using monthly CPI; annual is the year-end value divided by BEA population.

The monthly (real per capita) dividend return is the difference between the value weighted return and capital return on the NYSE+AMEX; this times the preceding month's market capitalization is converted to a real ($1996) monthly dividend series using the monthly CPI; annual is this series aggregated over the year divided by the BEA population.

Monthly (real per capita) stock return is value weighted return on NYSE+AMEX converted to real using the CPI; annual geometric return is this return cumulated over the year.

Annual quadratic variation is the sum of the monthly squared real geometric returns.
Log Real Per Capita Stock Market Value

Log Real Per Capita Stock Market Dividend

Log Real Per Capita Consumption, Measured at Annual Frequency

Real Annual Geometric Stock Market Returns

Log Quadratic Variation of Real Stock Market Returns
Cointegrating Relationships

\[ p^a_{dt} - d^a_t = I(0) \quad \text{Well documented in the literature} \]

\[ d^a_t - c^a_t = I(0) \quad \text{Verified with a reduced rank regression} \]

\[ c^a_t - c^a_{t-12} = I(0) \quad \text{Well documented in the literature} \]
Data to be Confronted by the Model

Used for estimation

\[ y_t = \begin{pmatrix} 
    d_t^a - c_t^a \\
    c_t^a - c_{t-12}^a \\
    p_{dt}^a - d_t^a \\
    r_{dt}^a 
\end{pmatrix} \]

Used for model validation

\[ y_t = \begin{pmatrix} 
    d_t^a - c_t^a \\
    c_t^a - c_{t-12}^a \\
    p_{dt}^a - d_t^a \\
    r_{dt}^a \\
    q_t^a 
\end{pmatrix} \]

\[ t = 12, 24, 36, \ldots, 12 \times 72 \]
Model Solution Method

Observation
We use a simulation estimator which means that a long simulation of the state vector \( \{\hat{u}_t\}_{t=1}^N \) will be available for each value of the parameters

\[
\rho(1) = (\mu_c, \lambda_{cs}, \rho_s, \lambda_{sx}, \rho_x, b_{cc}, b_{ss}, b_{xx}, \psi, \rho_\sigma, \mu_{dc})
\]

considered during estimation. Since the simulation is available, one might as well use it to solve the model.

The Relevant Fact
An unconditional expectation of a random variable of the form \( g(u_{t+1}, u_t) \) can be computed from a simulation as

\[
\mathcal{E}(g) \doteq \frac{1}{N} \sum_{t=1}^{N} g(\hat{u}_{t+1}, \hat{u}_t)
\]

to any desired degree of accuracy by taking \( N \) sufficiently large.
Model Solution Method

The Problem
Given a value of the parameters of the state dynamics

\[ \rho(1) = (\mu_c, \lambda_{cs}, \rho_s, \lambda_{sx}, \rho_x, b_{cc}, b_{ss}, b_{xx}, \Psi, \rho_\sigma, \mu_{dc}) \]

and a value of the parameters of the utility function

\[ \rho(2) = (\delta, \gamma, \psi) \]

find the pricing function \( v_c(u) \) that solves

\[ \mathcal{E}\{ v_c(u_t) - M(u_{t+1}, u_t)[1 + v_c(u_{t+1})] \exp(\Delta c_{t+1}) | u_t = u \} = 0 \]

for all \( u \in \mathbb{R}^3 \), where

\[ M(u_{t+1}, u_t) = \delta^\theta \exp[-(\theta/\psi)\Delta c_{t+1} + (\theta - 1)r_c(u_{t+1}, u_t)] \]

\[ r_c(u_{t+1}, u_t) = \log \left[ \frac{1 + v_c(u_{t+1})}{v_c(u_t)} \exp(\Delta c_{t+1}) \right] . \]
Model Solution Method

GMM Solution Strategy

Assume $v_c(u)$ is adequately approximated by a quadratic

$$v_c(u) = a_0 + a_1 u + u'A_2 u$$

put

$$g(u_{t+1}, u_t) = \left\{ v_c(u_t) - M(u_{t+1}, u_t)[1+v_c(u_{t+1})]\exp(\Delta c_{t+1}) \right\} Z_t$$

where $Z_t = [1, u_t', \text{vech}'(u_t, u_t)]'$ and solve the unconditional moment conditions

$$0 = \mathcal{E}g = \frac{1}{N} \sum_{t=1}^{N} g(\tilde{u}_{t+1}, \tilde{u}_t)$$

for $\alpha = [a_0, a_1', \text{vech}'(A_2)]$

GMM or NL2SLS software can be used to solve this system.

Similarly for $v_d(u)$, and $r_f(u)$. 
Estimation Method

Efficient Method of Moments (EMM) with a one lag VAR auxiliary model. With this choice of auxiliary model, the estimator is asymptotically equivalent to the estimator proposed by Tony Smith (1993).
Ex Ante Risk Free Rate

- The ex ante real rate risk free rate of interest is not directly observable but any reasonable asset pricing model must accommodate the evidence that the risk free rate $r_{ft}$ is about 0.896 percent per annum with low volatility (Campbell, 2002).

- We impose this restriction on $E(r_{ft})$ by imposing

  \[ \frac{1}{N} \sum_{t=1}^{n} r_{ft} = 0.000743618 \pm 0.0004167 \]

  on the monthly simulation.

- It binds. Without it, the estimation method chooses a risk free rate on the same order of magnitude as the real stock return. With it, estimates imply a reasonable equity premium.
Models Considered

**SRR, LRR, HAB**
Selected parameters calibrated, remainder estimated with EMM.

**Auxiliary Model**
The auxiliary model is an uncoupled VAR with one lag fit to the data without quadratic variation.

**Model Evaluation**
When evaluating the performance of Models SRR, LRR, and HAB, quadratic variation is taken into account.
### Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SRR Model</th>
<th>LRR Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_c$</td>
<td>0.002178</td>
<td>0.001921</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>0.9892</td>
<td>0.9583</td>
</tr>
<tr>
<td>$\lambda_{sx}$</td>
<td>2.5004</td>
<td>16.9346</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>0.9871</td>
<td>0.9871</td>
</tr>
<tr>
<td>$b_{cc}$</td>
<td>0.1432</td>
<td>c</td>
</tr>
<tr>
<td>$b_{ss}$</td>
<td>0.8188</td>
<td>0.2788</td>
</tr>
<tr>
<td>$b_{xx}$</td>
<td>0.1100</td>
<td>c</td>
</tr>
<tr>
<td>$\psi_{cc}$</td>
<td>0.00646</td>
<td>0.0034</td>
</tr>
<tr>
<td>$\psi_{cs}$</td>
<td>0.00146</td>
<td>0.01746</td>
</tr>
<tr>
<td>$\psi_{ss}$</td>
<td>0.02487</td>
<td>0.00809</td>
</tr>
<tr>
<td>$\psi_{xx}$</td>
<td>0.000120</td>
<td>c</td>
</tr>
<tr>
<td>$\rho_\sigma$</td>
<td>0.9866</td>
<td>0.0011</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.997488</td>
<td>0.999566</td>
</tr>
<tr>
<td>$\theta$</td>
<td>-195.79</td>
<td>-12.2843</td>
</tr>
<tr>
<td>$\psi$</td>
<td>2.00</td>
<td>c</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>98.8969</td>
<td>7.1421</td>
</tr>
<tr>
<td>$\mu_{dc}$</td>
<td>-3.3965</td>
<td>-3.3857</td>
</tr>
</tbody>
</table>

$\chi^2(5) = 41.051 (0.9e-7)$  
$\chi^2(3) = 10.501 (0.0148)$
## Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_c$</td>
<td>0.001937</td>
<td>0.000334</td>
</tr>
<tr>
<td>$\psi_{cc}$</td>
<td>0.003578</td>
<td>0.013025</td>
</tr>
<tr>
<td>$\psi_{cs}$</td>
<td>-0.003329</td>
<td>0.014111</td>
</tr>
<tr>
<td>$\psi_{ss}$</td>
<td>0.002187</td>
<td>0.000290</td>
</tr>
<tr>
<td>$\rho_h$</td>
<td>0.9889</td>
<td>0.0099</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.9954</td>
<td>0.0042</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.0357</td>
<td>2.1325</td>
</tr>
<tr>
<td>$\mu_{dc}$</td>
<td>-2.8237</td>
<td>0.0658</td>
</tr>
</tbody>
</table>

$\chi^2(5) = 14.476$ (0.0129)
### Unconditional Moments, LRR

<table>
<thead>
<tr>
<th>Metric</th>
<th>Formula</th>
<th>Observed</th>
<th>Predicted-LRR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log dividend consumption ratio</td>
<td>$d_t^a - c_t^a$</td>
<td>Mean: -3.399, Std Dev: 0.162</td>
<td>Mean: -3.384, Std Dev: 0.154</td>
</tr>
<tr>
<td>Consumption growth (% Per Year)</td>
<td>$100(c_t^a - c_{t-12}^a)$</td>
<td>Mean: 1.95, Std Dev: 2.24</td>
<td>Mean: 2.34, Std Dev: 2.36</td>
</tr>
<tr>
<td>Price dividend ratio</td>
<td>$\exp(v_{dt}^a)$</td>
<td>Mean: 28.24, Std Dev: 12.08</td>
<td>Mean: 27.47, Std Dev: 8.01</td>
</tr>
<tr>
<td>Return (% Per Year), dividend</td>
<td>$100 \times r_{dt}^a$</td>
<td>Mean: 6.02, Std Dev: 19.29</td>
<td>Mean: 6.29, Std Dev: 16.00</td>
</tr>
<tr>
<td>100 × √Quadratic variation</td>
<td>$100 \times \text{std}_t^a$</td>
<td>Mean: 16.69, Std Dev: 09.32</td>
<td>Mean: 14.95, Std Dev: 8.21</td>
</tr>
<tr>
<td>Risk free rate (% Per Year)</td>
<td>$100 \times r_{ft}^a$</td>
<td>Mean: 0.78, Std Dev: 1.35</td>
<td></td>
</tr>
<tr>
<td>Return (% Per Year), consumption</td>
<td>$100 \times r_{ct}^a$</td>
<td>Mean: 2.34, Std Dev: 3.95</td>
<td></td>
</tr>
<tr>
<td>Equity premium (% Per Year)</td>
<td>$100 \times r_{dt}^a - r_{ft}^a$</td>
<td>Mean: 5.51, Std Dev: 16.09</td>
<td></td>
</tr>
</tbody>
</table>
### Unconditional Moments, HAB

<table>
<thead>
<tr>
<th></th>
<th>Observed</th>
<th>Predicted-HAB</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Log dividend consumption ratio</strong> $d_t^a - c_t^a$</td>
<td>-3.399</td>
<td>-3.385</td>
</tr>
<tr>
<td><strong>Consumption growth(%) Per Year</strong> $100(\times c_t^a - c_{t-12}^a)$</td>
<td>1.95</td>
<td>2.35</td>
</tr>
<tr>
<td><strong>Price dividend ratio</strong> $\exp(v_{dt}^a)$</td>
<td>28.24</td>
<td>29.08</td>
</tr>
<tr>
<td><strong>Return(%) Per Year, dividend</strong> $100 \times r_{dt}^a$</td>
<td>6.02</td>
<td>6.22</td>
</tr>
<tr>
<td><strong>100 \times \sqrt{Quadratic variation}</strong> $100 \times \text{std}_{t}^a$</td>
<td>16.69</td>
<td>15.56</td>
</tr>
<tr>
<td><strong>Risk free rate (%) Per Year</strong> $100 \times r_{ft}^a$</td>
<td>1.23</td>
<td>1.23</td>
</tr>
<tr>
<td><strong>Return(%) Per Year, consumption</strong> $100 \times r_{ct}^a$</td>
<td>6.29</td>
<td>6.29</td>
</tr>
<tr>
<td><strong>Equity premium(%) Per Year</strong> $100 \times r_{dt}^a - r_{ft}^a$</td>
<td>4.99</td>
<td>4.99</td>
</tr>
</tbody>
</table>
One Step Ahead Forecasts, LRR Model

Log of Dividend Consumption Ratio

Consumption Growth

Log of Price Dividend Ratio

Annual Equity Returns

Equity Returns Volatility
One Step Ahead Forecasts, HAB Model
### Average Put and Call Prices on the Stock

<table>
<thead>
<tr>
<th>Strike/Underlying:</th>
<th>0.99</th>
<th>1.00</th>
<th>1.01</th>
<th>0.99</th>
<th>1.00</th>
<th>1.01</th>
<th>0.99</th>
<th>1.00</th>
<th>1.01</th>
<th>0.99</th>
<th>1.00</th>
<th>1.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expiration (mo)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2.19</td>
<td>1.65</td>
<td>1.22</td>
<td>1.38</td>
<td>1.85</td>
<td>2.41</td>
<td>2.21</td>
<td>1.67</td>
<td>1.27</td>
<td>1.44</td>
<td>1.89</td>
<td>2.50</td>
</tr>
<tr>
<td>2</td>
<td>2.79</td>
<td>2.28</td>
<td>1.84</td>
<td>2.18</td>
<td>2.67</td>
<td>3.23</td>
<td>2.83</td>
<td>2.30</td>
<td>1.88</td>
<td>2.28</td>
<td>2.75</td>
<td>3.33</td>
</tr>
<tr>
<td>3</td>
<td>3.25</td>
<td>2.75</td>
<td>2.31</td>
<td>2.83</td>
<td>3.33</td>
<td>3.89</td>
<td>3.30</td>
<td>2.78</td>
<td>2.35</td>
<td>2.97</td>
<td>3.45</td>
<td>4.01</td>
</tr>
<tr>
<td>4</td>
<td>3.61</td>
<td>3.13</td>
<td>2.69</td>
<td>3.38</td>
<td>3.89</td>
<td>4.45</td>
<td>3.68</td>
<td>3.17</td>
<td>2.72</td>
<td>3.56</td>
<td>4.04</td>
<td>4.59</td>
</tr>
<tr>
<td>5</td>
<td>3.93</td>
<td>3.45</td>
<td>3.01</td>
<td>3.88</td>
<td>4.40</td>
<td>4.96</td>
<td>4.00</td>
<td>3.49</td>
<td>3.04</td>
<td>4.10</td>
<td>4.58</td>
<td>5.13</td>
</tr>
<tr>
<td>6</td>
<td>4.21</td>
<td>3.73</td>
<td>3.30</td>
<td>4.34</td>
<td>4.86</td>
<td>5.42</td>
<td>4.29</td>
<td>3.78</td>
<td>3.33</td>
<td>4.61</td>
<td>5.09</td>
<td>5.63</td>
</tr>
<tr>
<td>7</td>
<td>4.46</td>
<td>3.99</td>
<td>3.56</td>
<td>4.76</td>
<td>5.29</td>
<td>5.85</td>
<td>4.55</td>
<td>4.05</td>
<td>3.59</td>
<td>5.08</td>
<td>5.57</td>
<td>6.11</td>
</tr>
<tr>
<td>8</td>
<td>4.69</td>
<td>4.23</td>
<td>3.80</td>
<td>5.15</td>
<td>5.68</td>
<td>6.25</td>
<td>4.79</td>
<td>4.29</td>
<td>3.83</td>
<td>5.54</td>
<td>6.03</td>
<td>6.56</td>
</tr>
<tr>
<td>9</td>
<td>4.90</td>
<td>4.44</td>
<td>4.01</td>
<td>5.54</td>
<td>6.07</td>
<td>6.64</td>
<td>5.01</td>
<td>4.51</td>
<td>4.06</td>
<td>5.98</td>
<td>6.48</td>
<td>7.01</td>
</tr>
<tr>
<td>10</td>
<td>5.08</td>
<td>4.63</td>
<td>4.21</td>
<td>5.92</td>
<td>6.45</td>
<td>7.02</td>
<td>5.21</td>
<td>4.72</td>
<td>4.26</td>
<td>6.41</td>
<td>6.90</td>
<td>7.44</td>
</tr>
<tr>
<td>11</td>
<td>5.25</td>
<td>4.80</td>
<td>4.38</td>
<td>6.28</td>
<td>6.82</td>
<td>7.39</td>
<td>5.40</td>
<td>4.90</td>
<td>4.45</td>
<td>6.81</td>
<td>7.31</td>
<td>7.84</td>
</tr>
<tr>
<td>12</td>
<td>5.40</td>
<td>4.96</td>
<td>4.54</td>
<td>6.64</td>
<td>7.19</td>
<td>7.76</td>
<td>5.57</td>
<td>5.07</td>
<td>4.62</td>
<td>7.22</td>
<td>7.71</td>
<td>8.24</td>
</tr>
</tbody>
</table>
### Average Put and Call Prices on Consumption

<table>
<thead>
<tr>
<th>Strike/Underlying:</th>
<th>0.99</th>
<th>1.00</th>
<th>1.01</th>
<th>0.99</th>
<th>1.00</th>
<th>1.01</th>
<th>0.99</th>
<th>1.00</th>
<th>1.01</th>
<th>0.99</th>
<th>1.00</th>
<th>1.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expiration (mo)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.38</td>
<td>0.31</td>
<td>0.24</td>
<td>0.12</td>
<td>0.14</td>
<td>0.18</td>
<td>0.59</td>
<td>0.51</td>
<td>0.44</td>
<td>0.14</td>
<td>0.16</td>
<td>0.19</td>
</tr>
<tr>
<td>2</td>
<td>0.59</td>
<td>0.51</td>
<td>0.44</td>
<td>0.17</td>
<td>0.19</td>
<td>0.21</td>
<td>0.69</td>
<td>0.62</td>
<td>0.55</td>
<td>0.13</td>
<td>0.16</td>
<td>0.19</td>
</tr>
<tr>
<td>3</td>
<td>0.79</td>
<td>0.71</td>
<td>0.63</td>
<td>0.20</td>
<td>0.22</td>
<td>0.24</td>
<td>0.85</td>
<td>0.78</td>
<td>0.71</td>
<td>0.14</td>
<td>0.16</td>
<td>0.19</td>
</tr>
<tr>
<td>4</td>
<td>0.98</td>
<td>0.90</td>
<td>0.82</td>
<td>0.23</td>
<td>0.25</td>
<td>0.27</td>
<td>1.01</td>
<td>0.93</td>
<td>0.86</td>
<td>0.14</td>
<td>0.16</td>
<td>0.19</td>
</tr>
<tr>
<td>5</td>
<td>1.16</td>
<td>1.08</td>
<td>1.00</td>
<td>0.25</td>
<td>0.27</td>
<td>0.29</td>
<td>1.16</td>
<td>1.08</td>
<td>1.01</td>
<td>0.14</td>
<td>0.16</td>
<td>0.19</td>
</tr>
<tr>
<td>6</td>
<td>1.34</td>
<td>1.26</td>
<td>1.18</td>
<td>0.28</td>
<td>0.29</td>
<td>0.31</td>
<td>1.31</td>
<td>1.23</td>
<td>1.16</td>
<td>0.14</td>
<td>0.16</td>
<td>0.18</td>
</tr>
<tr>
<td>7</td>
<td>1.51</td>
<td>1.43</td>
<td>1.35</td>
<td>0.30</td>
<td>0.31</td>
<td>0.33</td>
<td>1.46</td>
<td>1.38</td>
<td>1.30</td>
<td>0.14</td>
<td>0.16</td>
<td>0.18</td>
</tr>
<tr>
<td>8</td>
<td>1.69</td>
<td>1.60</td>
<td>1.52</td>
<td>0.32</td>
<td>0.33</td>
<td>0.35</td>
<td>1.61</td>
<td>1.53</td>
<td>1.45</td>
<td>0.14</td>
<td>0.16</td>
<td>0.18</td>
</tr>
<tr>
<td>9</td>
<td>1.86</td>
<td>1.77</td>
<td>1.69</td>
<td>0.34</td>
<td>0.35</td>
<td>0.37</td>
<td>1.76</td>
<td>1.68</td>
<td>1.60</td>
<td>0.14</td>
<td>0.16</td>
<td>0.18</td>
</tr>
<tr>
<td>10</td>
<td>2.03</td>
<td>1.94</td>
<td>1.86</td>
<td>0.36</td>
<td>0.37</td>
<td>0.39</td>
<td>1.91</td>
<td>1.82</td>
<td>1.74</td>
<td>0.14</td>
<td>0.16</td>
<td>0.17</td>
</tr>
<tr>
<td>11</td>
<td>2.19</td>
<td>2.11</td>
<td>2.03</td>
<td>0.38</td>
<td>0.40</td>
<td>0.41</td>
<td>2.05</td>
<td>1.97</td>
<td>1.89</td>
<td>0.14</td>
<td>0.15</td>
<td>0.17</td>
</tr>
<tr>
<td>12</td>
<td>2.36</td>
<td>2.27</td>
<td>2.19</td>
<td>0.41</td>
<td>0.42</td>
<td>0.44</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Average Put and Call Prices on Wealth

<table>
<thead>
<tr>
<th>Strike/Underlying:</th>
<th>0.99</th>
<th>1.00</th>
<th>1.01</th>
<th>0.99</th>
<th>1.00</th>
<th>1.01</th>
<th>0.99</th>
<th>1.00</th>
<th>1.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expiration (mo)</td>
<td>Call</td>
<td>Put</td>
<td>Call</td>
<td>Put</td>
<td>Call</td>
<td>Put</td>
<td>Call</td>
<td>Put</td>
<td>Call</td>
</tr>
<tr>
<td>1</td>
<td>1.18</td>
<td>0.43</td>
<td>0.10</td>
<td>0.11</td>
<td>0.36</td>
<td>1.03</td>
<td>2.26</td>
<td>1.72</td>
<td>1.32</td>
</tr>
<tr>
<td>2</td>
<td>1.35</td>
<td>0.63</td>
<td>0.22</td>
<td>0.22</td>
<td>0.50</td>
<td>1.09</td>
<td>2.90</td>
<td>2.38</td>
<td>1.96</td>
</tr>
<tr>
<td>3</td>
<td>1.51</td>
<td>0.80</td>
<td>0.35</td>
<td>0.32</td>
<td>0.61</td>
<td>1.15</td>
<td>3.39</td>
<td>2.88</td>
<td>2.44</td>
</tr>
<tr>
<td>4</td>
<td>1.65</td>
<td>0.95</td>
<td>0.47</td>
<td>0.40</td>
<td>0.70</td>
<td>1.21</td>
<td>3.78</td>
<td>3.27</td>
<td>2.82</td>
</tr>
<tr>
<td>5</td>
<td>1.79</td>
<td>1.09</td>
<td>0.59</td>
<td>0.48</td>
<td>0.78</td>
<td>1.27</td>
<td>4.12</td>
<td>3.61</td>
<td>3.16</td>
</tr>
<tr>
<td>6</td>
<td>1.92</td>
<td>1.23</td>
<td>0.71</td>
<td>0.55</td>
<td>0.85</td>
<td>1.32</td>
<td>4.41</td>
<td>3.91</td>
<td>3.45</td>
</tr>
<tr>
<td>7</td>
<td>2.05</td>
<td>1.36</td>
<td>0.82</td>
<td>0.62</td>
<td>0.92</td>
<td>1.38</td>
<td>4.68</td>
<td>4.18</td>
<td>3.73</td>
</tr>
<tr>
<td>8</td>
<td>2.18</td>
<td>1.48</td>
<td>0.93</td>
<td>0.68</td>
<td>0.98</td>
<td>1.43</td>
<td>4.93</td>
<td>4.43</td>
<td>3.97</td>
</tr>
<tr>
<td>9</td>
<td>2.30</td>
<td>1.61</td>
<td>1.04</td>
<td>0.75</td>
<td>1.05</td>
<td>1.48</td>
<td>5.15</td>
<td>4.66</td>
<td>4.21</td>
</tr>
<tr>
<td>10</td>
<td>2.42</td>
<td>1.72</td>
<td>1.15</td>
<td>0.81</td>
<td>1.12</td>
<td>1.54</td>
<td>5.36</td>
<td>4.87</td>
<td>4.42</td>
</tr>
<tr>
<td>11</td>
<td>2.53</td>
<td>1.84</td>
<td>1.26</td>
<td>0.88</td>
<td>1.18</td>
<td>1.60</td>
<td>5.55</td>
<td>5.06</td>
<td>4.61</td>
</tr>
<tr>
<td>12</td>
<td>2.65</td>
<td>1.95</td>
<td>1.37</td>
<td>0.96</td>
<td>1.26</td>
<td>1.66</td>
<td>5.73</td>
<td>5.24</td>
<td>4.78</td>
</tr>
</tbody>
</table>
Predictability Projections: Price Dividend Ratios

<table>
<thead>
<tr>
<th></th>
<th>Observed Coef</th>
<th>SE</th>
<th>Predicted LRR</th>
<th>Predicted HAB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>3.4168</td>
<td>0.0055</td>
<td>3.2520</td>
<td>2.5046</td>
</tr>
<tr>
<td>$\Delta c_t$</td>
<td>-0.3021</td>
<td>0.1744</td>
<td>1.0716</td>
<td>9.8700</td>
</tr>
<tr>
<td>$\Delta c_{t-24}$</td>
<td>-1.0560</td>
<td>0.1926</td>
<td>0.0383</td>
<td>5.5366</td>
</tr>
<tr>
<td>$\Delta c_{t-48}$</td>
<td>-0.6209</td>
<td>0.1925</td>
<td>0.2521</td>
<td>5.8474</td>
</tr>
<tr>
<td>$\Delta c_{t-60}$</td>
<td>-0.3480</td>
<td>0.1926</td>
<td>-0.0176</td>
<td>5.2568</td>
</tr>
<tr>
<td>$\Delta c_{t-72}$</td>
<td>-0.1263</td>
<td>0.1926</td>
<td>-0.0442</td>
<td>3.7064</td>
</tr>
<tr>
<td>$\Delta c_{t-84}$</td>
<td>-0.2006</td>
<td>0.1744</td>
<td>-0.0857</td>
<td>4.7340</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.036</td>
<td></td>
<td>0.011</td>
<td>0.529</td>
</tr>
</tbody>
</table>

Notes: Shown above are the linear projections of the log price dividend ratio, $v_d$ on contemporaneous and five annual lags of log consumption growth $\Delta c$. The period for the observed projection is 1935–2001. The predicted values are from long simulations from the Long Run Risks $LRR$ Model and the Habit Persistence $HAB$ Model.
Long Horizon Predictability Projections:
Cumulative Future Return on the Price Dividend Ratio

<table>
<thead>
<tr>
<th>Horizon(Years)</th>
<th>Observed</th>
<th>LRR</th>
<th>HAB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.038</td>
<td>0.088</td>
<td>0.112</td>
</tr>
<tr>
<td>2</td>
<td>0.060</td>
<td>0.129</td>
<td>0.207</td>
</tr>
<tr>
<td>3</td>
<td>0.071</td>
<td>0.172</td>
<td>0.289</td>
</tr>
<tr>
<td>4</td>
<td>0.071</td>
<td>0.202</td>
<td>0.362</td>
</tr>
<tr>
<td>5</td>
<td>0.070</td>
<td>0.229</td>
<td>0.423</td>
</tr>
</tbody>
</table>

Notes: The table shows $R^2$'s from projections of cumulative annual geometric returns for 1,2,...,5 years ahead onto the log price dividend ratio for the observed data, 1935–2001, and for long simulations from the Long Run Risks **LRR** Model and the Habit Persistence **HAB** Model.